DYNAMIC ALLOCATION OF AIRPORT SECURITY RESOURCES:
A GLOBAL OPTIMIZATION APPROACH

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Plan of Presentation

- Introduction
- System Analysis
- Dynamic Network Modelling
- Performance indexes
- Global Optimization Problem
- Solution Approach
- Application
- Conclusions
In the post 9-11 era, airport security has become worldwide not only a critical issue but also a very costly one for airports as well as for passengers and airlines.

Airports have been induced to invest massively in new installations, equipment and workforce to implement the strengthened new security regulations. The cost for the public has been a bitter mix of increased airport taxes, new inconveniences and lengthened delays.

In this communication, the problem of optimizing the allocation of equipments and work teams to control flows of passengers in an airport terminal is considered. The main objective is to minimize the possibility of dangerous situations inside the passenger terminal including dubious passenger being admitted on board an aircraft, but another objective is to insure a minimum quality of service to passengers.
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Parcours des passagers dans un aéroport
Here passengers terminals are represented by sets of passengers and luggage processing machines located on a network whose flows adopt the different available paths between terminal accesses and boarding gates. A characteristic of airport passengers flows is their variability according to the airport flight departure-arrival schedules.

A general mesoscopic modelling approach is introduced for passengers flows in an airport terminal which should be compatible with the formulation of relevant short term optimization problems for the allocation of available security equipment and staff.

Different instances of global optimization can be proposed but when considering that upstream (check-in) and downstream (boarding) processes follow specified operations rules, the resulting optimization problem concentrates on the optimization of security operations.
The number of passengers arriving to the terminal by entry \( j \) during time period \( n \) ([\( t_0 + n \Delta t \), \( t_0 + (n+1) \Delta t \)], to take a given flight \( k \) is given by:

\[
d_{kj}(n) = D_k \int_{t_0 + n \Delta t}^{t_0 + (n+1) \Delta t} f dp_{kj}(t_{dk}, t) \, dt \quad n_{ak} < n < n_{dk} \quad j \in E_k, k \in K
\]  

(1)

where \( f dp_{kj}(t_{dk}, t) \, dt \) is the probability density of arrivals for flight \( k \) at entry \( j \).

Here \( K \) is the set of considered flights, \( E_k \) is the set of entries for flight \( k \) and \( D_k \) is total demand for flight \( k \), \( n_{ak} \) is the time period before which no passenger for flight \( k \) appears and is the last time
Figure 1. General passenger control structure at airports
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The circulation of departing passengers between entry points and boarding gates can be represented by a network of processing units based on a simple graph $G = [X, U]$.

This simple graph $G$ is composed of five classes of arcs and seven classes of nodes:
- entry arcs $i$ from entry nodes ($e(i)$ is the origin node of arc $i$);
- circulation arcs $v$ from check-in nodes to control queuing arcs or from check-in nodes to circulation arcs (in both cases $a(v)$ is the origin node of arc $v$), circulation arcs $u$ from control nodes to circulation arcs ($c(u)$ is the origin node of arc $u$) and from
- circulation arcs to other circulation arcs;
- queuing arcs at security control positions from circulation arcs to other circulation arcs,
- queuing arcs at check-in points from circulation arcs to other circulation arcs,
- queuing arcs at boarding gates ($b(i)$ is the boarding gate associated to queuing arc $i$).

$$U = U_{\text{ent}} \cup U_{\text{cir}} \cup U_{\text{cnt}} \cup U_{\text{che}} \cup U_{\text{brd}}$$
$$X = X_{\text{ent}} \cup X_{\text{cor}} \cup X_{\text{che}} \cup X_{\text{que}} \cup X_{\text{cnt}} \cup X_{\text{cir}} \cup X_{\text{brd}}$$
Figure 2. A network representation of passenger departure processing
Flow dynamics for entry circulation arcs

\[ \phi_i(n) = \sum_k f_{ik}(n) \]

with

\[
\begin{align*}
  f_{ik}(n) &= 0 & & \text{if } n \not\in N_k \quad \text{otherwise} \\
  f_{ik}(n) &= \max\{0, (1 - \lambda_i / (V_i \Delta t)) f_{ik}(n-1) \} + \alpha_{kh} d_{kh}(n) & & \text{if } \delta_{ik} = 1 \\
  f_{ik}(n) &= \max\{0, (1 - \lambda_i / (V_i \Delta t)) f_{ik}(n-1) \} + (1 - \alpha_{kh}) d_{kh}(n) & & \text{if } \epsilon_{ik} = 1 \\
  \text{and } f_{ik}(n) &= \max\{0, (1 - \lambda_i / (V_i \Delta t)) f_{ik}(n-1) \} + d_{kh}(n) & & \text{if } \gamma_{ik} = 1 
\end{align*}
\]

where \( h = e(i) \), \( \delta_{ik} = 1 \) if arc \( i \) leads passengers for flight \( k \) from the entry node \( e(i) \) to a check-in node for flight \( k \), \( \delta_{ik} = 0 \) otherwise, \( \epsilon_{ik} = 1 \) if arc \( i \) leads passengers for flight \( k \) directly to a control point, \( \epsilon_{ik} = 0 \) otherwise, \( \gamma_{ik} = 1 \) if arc \( i \) leads correspondance passengers for flight \( k \) to a control position. We have the following coherence constraint for \( i \in U_{ent}, k \in K \):

\[ 0 \leq \delta_{ik} + \epsilon_{ik} + \gamma_{ik} \leq 1 \]
Flow dynamics for queuing at security control

\[
\phi_i(n) = \sum_k f_{ik}(n) = \max \{ 0, \phi_i(n-1) - \Phi_{cnt} \Delta t \ y_h(n-1) \} + \sum_{u \in \Gamma_i^{-1}} \omega_u(n-1)
\]

where: \( h = c(i) \) and \( \omega_u(n-1) = \phi_u(n-1) \lambda_u / (V_u \Delta t) \ u \in \Gamma_i^{-1} \)

Let:

\[
\delta f_{ik}(n) = \sum_{u \in \Gamma_i^{-1}} \omega_{iku}(n-1)
\]

then:

\[
f_{ik}(n) = \delta f_{ik}(n) = 0 \text{ if } i \not\in ch(k) \text{ and otherwise } f_{ik}(n) = \delta f_{ik}(n) \text{ if } f_i(n-1) \geq \Delta t \ y_h \\
\text{or } f_{ik}(n) = \max \{ 0, f_{ik}(n-1) - \Delta f_{ik}(n) \} + \delta f_{ik}(n) \text{ if } f_i(n-1) > \Phi \Delta t \ y_h
\]
\[ nc_{\min}(i, k) = q \quad \text{if} \quad \delta F_{ik}(p) = 0 \quad \forall p < q - 1 \quad \text{and} \quad \delta F_{ik}(q) > 0 \quad p, q \in \mathbb{N} \]

\[ nc_{MIN}(i, n) = \min_{k \in ech(k)} nc_{\min}(i, k) \]

and \( m(i, n) \) such that:

\[ m(i, n) \leq n \quad \text{and} \quad \sum_{n' = nc_{MIN}(i, n)}^{m} \sum_{k \in ech(k)} \delta F_{ik}(n') \leq \Phi \Delta t y_h(n - 1) < \sum_{n' = nc_{MIN}(i, n)}^{m-1} \sum_{k \in ech(k)} \delta F_{ik}(n') \]

Now:

\[ \text{if} \quad \sum_{n' = nc_{MIN}(i, n)}^{m} \sum_{k \in ech(k)} \delta F_{ik}(n') \leq \Phi \Delta t y_h(n - 1) \quad \text{then} \quad nc_{\min}(i, k) = m \]

and

\[ \Delta f_{ik}(n) = \sum_{n' = nc_{MIN}(i, n)}^{nc_{\min}(i, k)} \delta F_{ik}(n') \]

\[ \text{if} \quad \Phi \Delta t y_h < \sum_{n' = nc_{MIN}(i, n)}^{m - 1} \sum_{k \in ech(k)} \delta F_{ik}(n') \quad \text{then} \quad nc_{\min}(i, k) = m - 1 \]

and

\[ \Delta f_{ik}(n) = \sum_{n' = nc_{MIN}(i, n)}^{nc_{\min}(i, k)} \delta F_{ik}(n') + \left( \sum_{n' = nc_{MIN}(i, n)}^{m - 1} \sum_{k \in ech(k)} \delta F_{ik}(n') - \Phi \Delta t y_h(n - 1) \right) \left( f_{ik}(n - 1) / f_i(n - 1) \right) \]
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PERFORMANCE EVALUATION

The following performance issues are of main interest:

- Security of passengers within the airport terminal and boarding an aircraft;

- Comfort for passengers through generalized waiting times and length of paths between entry points and boarding gates;

- Operating costs for airlines and airport security systems.
**Passenger Comfort**

With respect to passenger comfort, total waiting times at check-in $i$ during a chosen time period $[n_s, n_e]$, are given by:

$$W_{che}(i, n_s, n_e) = \sum_{n=n_s}^{n_e} \phi_i(n-1) \quad n_s < n_e \quad i \in X_{che}$$

while the corresponding mean waiting times per passengers are estimated by:

$$\bar{W}_{che}(i, n_s, n_e) = \sum_{n=n_s}^{n_e} \phi_i(n-1) / \sum_{n=n_s}^{n_e} \sum_{u \in \Gamma_i^{-1}} \omega_u(n-1)$$

where the integers $\tau_s$ and $\tau_e$ are adjustable parameters.

Total waiting times for passengers of flight $k$ at control point $i$, are given by:

$$w_{cnt}(i, k, n_s, n_e) = \sum_{n=n_s}^{n_e} f_{ik}(n-1)$$

while the corresponding mean waiting times per passengers are estimated by:

$$\bar{w}_{cnt}(i, k, n_s, n_e) = \sum_{n=n_s}^{n_e} f_{ik}(n-1) / \sum_{n=n_s}^{n_e} \sum_{u \in \Gamma_i^{-1}} \omega_{ik}(n-1)$$
Efficiency at control stations

It is supposed that a team which has been assigned to a control position $j$ in a control station $c(i)$ at the $p^{th}$ shift of beginning time period $n_s(p,i)$ is changed at the end of time period $n_e(p,i)$ such that:

$$p_{\text{max}}(i) = \left[ (N_{e}(i) - N_{s}(i)) / n_{\text{min}}^w \right]$$

The control efficiency of the $p^{th}$ shift can be considered to be an increasing function $\rho$ of the ratio given by:

$$r_{\text{cnt}}(p,i) = (n_{e}(p,i) - n_{s}(p,i)) / n_{\text{max}}^w(i)$$

which varies from $r_{\text{min}}(i) = d_{\text{max}}^p / (n_{\text{max}}^w(i) \Phi \Delta t)$ to 1 and depicts the intensity of the control task.
Figure 3. Control efficiency as a function of task intensity
A global criterion to assess control efficiency during a whole day can be given by:

\[ C_{\text{cnt}}(y) = \sum_{i \in X_{\text{cnt}}} \sum_{p=1}^{p_{\text{max}}(i)} y_i(i, p) \left( \rho(r_{\text{cnt}}(p,i)) - \rho_{\text{max}} \right)^2 \]

other global indexes are:

\[
\rho_{\text{min}}^* = \min_{i \in X_{\text{cnt}}} \min_{p=1 \text{ to } p_{\text{max}}(i)} \rho(r_{\text{min}}(p,i))
\]

and

\[ |F_\epsilon| \text{ where } F_\epsilon = \bigcup_{i \in X_{\text{cnt}}} F_\epsilon(i) \text{ with } F_\epsilon(i) = \{ \text{shifts } m \text{ such as } \rho(r_{\text{min}}(i,p) \leq \rho_{\text{max}}(1 - \epsilon) \}
\]

with \( 0 < \epsilon < 1 \)
At the level of each control position $i$, we have the corresponding security performance indexes:

$$
C_{cnt}(i) = \sum_{p=1}^{p_{\text{max}}(i)} y_{shift}(i, p) (\rho(r_{\text{min}}(p, i)) - \rho_{\text{max}})^2
$$

and

$$
\rho^*_{\text{min}}(i) = \min_{p=1 \text{ to } p_{\text{max}}(i)} \rho(r_{\text{min}}(m, i)) \quad \text{and} \quad |F_{\varepsilon}(i)| \quad i \in X_{cnt} \quad 0 < \varepsilon < 1
$$
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Efficiency with respect to soft targets

Crowded areas such as queues for check-in, security control and boarding which present an increased density of humans represent a potential target. It is specially the case with check-in queues since passengers luggages have not been in general controlled beforehand. So it appears of real importance from this point of view to diminish queues at this stage. To diminish the size of the check-in queues different actions can be adopted: increased number of check-in positions, reprogramming of flights too close in time, distribution of check-in activities over different areas, extension of electronic ticketing or remote check-in..

Efficiency with special high security flights

Some flight, for different particular reasons may demand for particular treatment, including separated check-in counters, segregated paths and remote boarding. In this case, what may be more relevant at the check-in stage is to minimize its duration while control must be performed at a smaller rate.
From medium to short term decision making

Many medium term decisions must be taken to achieve an efficient management and control of passenger flows in an airport terminal: check-in counters position and size, security control positions and size, assignment and timing of flights to gates, assignment of operators to counters and security control positions.

Once medium term decisions defining a reference environment for short term decision making, efficient decisions relative to the assignment of control positions to passengers, the assignment of paths to passengers, staff shifts at control positions and deployment at check-in and security control positions must be determined.

Many candidate short term optimization problems can be formulated according to the assumed environment, the adopted constraints and considered objectives.
Short term optimal management of security control activities

\[
\min_{ch(k), k \in K, z_j(n), j \in X_{che}, y_{shift}(i, p), n_x(p, i), n_y(p, i), 1 \leq p \leq p_{\text{max}}(i), i \in X_{cnt}} \sum_{p = 1}^{p_{\text{max}}(i)} \sum_{i \in X_{cnt}} y_{shift}(i, p) \left( \rho(r_{\text{cnt}}(p, i)) - \rho_{\text{ml}} \right)^2
\]

with

\[
\sum_{n = N_x(t)}^{N_x(t) - \tau_x} \Phi_x(n - 1) \leq T_{\text{max}}^{\text{che}}(j) \sum_{n = N_x(t) - \tau_x}^{N_x(t)} \Delta t \ z_j(n - 1) \quad t \in \Gamma_{\text{eva}, j \in X_{che}}
\]

\[
\sum_{n = N_y(t)}^{N_y(t) - \tau_y} \Phi_y(n - 1) \leq T_{\text{max}}^{\text{cnt}}(i) \sum_{n = N_y(t) - \tau_y}^{N_y(t)} \Phi_{\text{cnt}} \Delta t \ y_{shift}(i, p) \delta_{i, p}^n \quad t \in \Gamma_{\text{eva}, i \in X_{cnt}}
\]

\[
\sum_{n = N_z(t)}^{N_z(t) - \tau_z} \Phi_z(n - 1) \leq T_{\text{max}}^{\text{brd}}(k) \sum_{n = N_z(t) - \tau_z}^{N_z(t)} \Phi_{\text{brd}} \Delta t \ x_k(n - 1) \quad t \in \Gamma_{\text{eva}, k \in X_{brd}}
\]

\[
\sum_{j \in X_{che}} z_j(n) e_{ja} + \sum_{k \in X_{brd}} x_k(n) \theta_{ka}^n \leq z_{\text{max}}^a \quad a \in A, n \in \{0, 1, \cdots, n_{\text{max}}\}
\]
with

\[
\begin{align*}
  z_j(n) &\in \{0, 1, \ldots, n_{ch}(j)\} \quad n \in \{0, 1, \ldots, n_{\text{max}}\}, j \in X, \\
  x_k(n) &\in \{0, 1, \ldots, n_{brd}(k)\} \quad n \in \{0, 1, \ldots, n_{\text{max}}\}, k \in X, \\
  n_e(p,i) &\geq n_s(p-1,i) \quad p \in \{1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{ch}}, \\
  n_{\text{min}}^w(i) &\leq n_e(p,i) - n_s(p,i) \leq n_{\text{max}}^w(i) \quad p \in \{0, 1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{ch}}, \\
  \sum_{i \in X_{\text{cnt}}} \sum_{p=1}^{p_{\text{max}}(i)} (n_e(p,i) - n_s(p,i)) y_{\text{shift}}(i,p) &\leq W_{\text{max}}, \\
  \sum_{i \in X_{\text{cnt}}} \sum_{p=1}^{p_{\text{max}}(i)} y_{\text{shift}}(i,p) \sigma_{i,p}^n &\leq y_{\text{max}} \quad n \in \{0, 1, \ldots, n_{\text{ma}}\}, \\
  n_e(p,i), n_s(p,i) &\in \{0, \ldots, n_{\text{max}}(i)\} \quad p \in \{0, 1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{cnt}}, \\
  y_{\text{shift}}(i,p) &\in \{0, 1, \ldots, n_{\text{pc}}(i)\} \quad p \in \{0, 1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{cnt}}.
\end{align*}
\]

and all the networks and queuing constraints.
Figure 4. General control structure
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A Solution Approach

The above optimization problem is highly complex since it is a large scale network-constrained problem with hybrid variables (operations parameters are taken as integer numbers while flows are taken as real numbers). No exact solution method with acceptable computation time is available while in theory, techniques such as Dynamic Programming, are compatible with its source-sink flow network structure, but its dimension is totally deterrent.

A global optimization approach based on sensitivity analysis should also lead to cumbersome simulation computations to calculate the different objective levels (control efficiency and mean delays) while a local one could lead to very sub-optimal solutions.

Here we consider that passengers and aircraft traffic at airports remain quite similar from one week to another, so that the adopted solution one week earlier can constitute an acceptable starting point for a search solution process.
If one week earlier no serious operational problem has appeared at the checking, control and boarding levels, it can be supposed that the paths assigned to each flight are maintained, $ch(k), k \in K$ that variable is fixed. Otherwise a new solution should be established for it.

Given then a previous solution:

$$\tilde{z}_j(n) \in \{0, 1, \ldots, n_{ch(j)}\} \quad n \in \{0, 1, \ldots, n_{\text{max}}\}, j \in X_{\text{che}}$$

$$\tilde{\gamma}_{\text{shift}}(i, p) \in \{0, 1, \ldots, n_{\text{pc}}(i)\} \quad p \in \{0, 1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{cnt}}$$

$$\tilde{n}_e(p, i), \tilde{n}_z(p, i) \quad p \in \{0, 1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{cnt}}$$

Each of the operating agent (each airlines if each one has her own check-in positions and the airport authority in charge of managing the control positions) can solve a local adaptation problem.
Local optimization by airlines

In the case of the airlines we can consider the following problems \((PQA_a)\):

\[
\min_{z_j(n), x_k(n)} \sum_n \left( \sum_{j \in X_{cha}} \left( a_{ja}(n) z_j(n) - \tilde{a}_{ja}(n) \tilde{z}_j(n) \right) + \sum_{k \in X_{brd}} \left( b_{ka}(n) x_k(n) - \tilde{b}_{ka}(n) \tilde{x}_k(n) \right) \right)^2 \theta_{k,a}^n
\]

with

\[
\sum_{j \in X_{cha}} z_j(n) \varepsilon_{ja}^n + \sum_{k \in X_{brd}} x_k(n) \theta_{ka}^n \leq z_{a,n}^{\max} \quad n \in \{0, 1, \ldots, n_{\max}\}
\]

\[
z_j(n) \in \{0, 1, \ldots, n_{ch}(j)\} \quad n \in \{0, 1, \ldots, n_{\max}\}, j \in X_{cha}
\]

\[
x_k(n) \in \{0, 1, \ldots, n_{brd}(k)\} \quad n \in \{0, 1, \ldots, n_{\max}\}, k \in X_{brd}
\]

where the \(a_{ja}(n), \tilde{a}_{ja}(n), b_{ka}\) and \(\tilde{b}_{ka}\) are weighting parameters established empirically. The, \(a_{ja}(n)\) can represent the expected number of passengers arriving at check in position \(j\) operated by airline \(a\) during time period \(i\), while \(\tilde{a}_{ja}(n)\) can represent the observed number of passengers arriving at check in position \(j\) during the same time period one week earlier and corrected taken into account the past performance with respect to subsequent constraints

\[
\tilde{a}_{ja}(n) = \max\{a_{ja}^0, a_{ja}(n)\} \left( \sum_{j \in X_{cha}, i \in T_j} (\widetilde{a}_{ji}(n) - T_{\max}(i) \Phi_{em} \Delta t \, \tilde{y}_{sh}(i, p) \, \tilde{\theta}_{i,p}^n) \right)
\]

and in the same way:

\[
\tilde{b}_{ka}(n) = \max\{b_{ka}^0, b_{ka}(n)\} \left( \sum_{k \in X_{brd}, i \in T_k} (\widetilde{b}_{ki}(n) - T_{\max}(k) \Phi_{brd} \Delta t \, x_k, \tilde{\theta}_{p}^n) \right)
\]
where the $\lambda_q(n)$ and $\mu_k(n)$, $n \in \{0,1,\ldots,n_{\text{max}}\}$, are positive parameters similar to Lagrange variables and where $a_{\text{min}}$ and $b_{\text{min}}$ are positive minimum values for $\tilde{a}_{ja}(n)$ and $\tilde{b}_{ka}$.

If the following conditions are satisfied:

$$
\sum_{j \in X_{ja}} \left| (\tilde{a}_{ja}(n)/a_{ja}(n)) \tilde{z}_j(n) \right| \varepsilon_{ja}^n + \sum_{k \in K_{ka}} \left| (\tilde{b}_{ka}(n)/b_{ka}(n)) \tilde{x}_k(n) \right| \theta_{ka}^n \leq z_a^{\text{max}} \quad n \in \{0,1,\ldots,n_{\text{max}}\}
$$

then a simple solution at check in position $j$ will be:

$$
z_j(n) = \left| (\tilde{a}_{ja}(n)/a_{ja}(n)) \tilde{z}_j(n) \right| \quad \text{with} \quad \varepsilon_{j,a}^n = 1
$$

$$
x_k(n) = \left| (\tilde{b}_{ka}(n)/b_{ka}(n)) \tilde{x}_k(n) \right| \quad \text{with} \quad \theta_{k,a}^n = 1
$$

otherwise the solution can be obtained using for instance a Branch and Bound approach.
Local assignment of control resources

A step towards an improved solution is at first to consider that the temporal pattern is maintained fixed and to improve intensities according to expected needs. In that case we can solve the surrogate problem \((PQC)\):

\[
\min_{y_{\text{shift}}(i,p)} \sum_{i \in X_{\text{cnt}}} \sum_{p=1}^{p_{\text{max}}(i)} (c_{ip} y_{\text{shift}}(i,p) - c_{ip} \hat{y}_{\text{shift}}(i,p))^2
\]

\[
\sum_{i \in X_{\text{cnt}}} \sum_{p=1}^{p_{\text{max}}(i)} (n_0(p,i) - \bar{n}(p,i)) y_{\text{shift}}(i,p) \leq W_{\text{max}}
\]

\[
\sum_{i \in X_{\text{cnt}}} \sum_{p=1}^{p_{\text{max}}(i)} y_{\text{shift}}(i,p) \tilde{\gamma}_{i,p} \leq y_{\text{max}} \quad n \in \{0, 1, \ldots, n_{\text{max}}\}
\]

\[
y_{\text{shift}}(i,p) \in \{0, 1, \ldots, n_{\text{pc}}(i)\} \quad p \in \{0, 1, \ldots, p_{\text{max}}(i)\}, i \in X_{\text{cnt}}
\]

Starting with the \(\hat{y}_{\text{shift}}\) solution, here also, a Branch and Bound solution approach can be implemented efficiently. \(\hat{y}_{\text{shift}}\) be the current solution.
Then in another step, the following problem (PLC) can be considered:

\[
\min_{\substack{n_e(p, i), n_z(p, i), 1 \leq p \leq P_{\text{max}}(i), i \in X_{\text{out}}} } \sum_{i \in X_{\text{out}}} \sum_{p=1}^{P_{\text{max}}(i)} (\hat{\gamma}_{\text{shift}}(i, p)(\rho(\gamma_{\text{cnt}}(p, i)) - \rho_{\text{max}}) (\partial \rho / \partial \gamma_p)(n_e(p, i) - n_z(p, i)) \]

with the constraints

\[
n_e(p, i) + n_z(p, i) \geq n_z(p-1, i) \quad p \in \{1, \ldots, P_{\text{max}}(i)\}, i \in X_{\text{cnt}}
\]

\[
n_{\text{min}}^w(i) \leq n_e(p, i) - n_z(p, i) \leq n_{\text{max}}^w(i) \quad p \in \{0, 1, \ldots, P_{\text{max}}(i)\}, i \in X_{\text{cnt}}
\]

\[
\sum_{i \in X_{\text{out}}} \sum_{p=1}^{P_{\text{max}}(i)} (n_e(p, i) - n_z(p, i)) \hat{\gamma}_{\text{shift}}(i, p) \leq W_{\text{max}}
\]

\[
n_e(p, i), n_z(p, i) \in \{0, \ldots, n_{\text{max}}(i)\} \quad p \in \{0, 1, \ldots, P_{\text{max}}(i)\}
\]

which is a linear integer problem which can be solved using some column generation technique.
Coordination

The coordination between the solution of the two problems is achieved by taking better into account the interactions between the two management subsystems so that an overall improved solution is obtained.

This is realized by performing at each step for a current partial solution \((PQA_a^{(h)}, PQC^{(h-1)} \text{ and } PLC^{(h-1)})\) and \((PQA_a^{(h)}, PQC^{(h)} \text{ and } PLC^{(h-1)})\) or a global solution \((PQA_a^{(h)}, PQC^{(h)} \text{ and } PLC^{(h)})\) an overall simulation of the system and then modifying the predicted flows and weights associated to the mean waiting time constraints.
The weight updating process can be such as:

\[
\text{If } \sum_{i \in X_{\text{cnt}}, j \in \Gamma_j} (\phi_i(n) - T_{\text{max}}(i) \Phi_{\text{cnt}} \Delta t \bar{y}_{\text{shift}}(i, p) \bar{\delta}_{i, p}) > 0 \text{ then } \\
\lambda_{\alpha j}^{(h)}(n) = \min \{ \lambda_{\text{max}}, \lambda_{\alpha j}^{(h-1)}(n)(1 + \sum_{i \in X_{\text{cnt}}, j \in \Gamma_j} (\phi_i(n) - T_{\text{max}}(i) \Phi_{\text{cnt}} \Delta t \bar{y}_{\text{shift}}(i, p) \bar{\delta}_{i, p})) \} \\
\text{If } \sum_{i \in X_{\text{cnt}}, j \in \Gamma_j} (\phi_i(n) - T_{\text{max}}(i) \Phi_{\text{cnt}} \Delta t \bar{y}_{\text{shift}}(i, p) \bar{\delta}_{i, p}) \leq 0 \text{ then } \lambda_{\alpha j}^{(h)}(n) = \lambda_{\alpha j}^{(h-1)}(n) \\
\text{If } \sum_{k \in X_{\text{brd}}} (\phi_k(n) - T_{\text{max}}(k) \Phi_{\text{brd}} \Delta t x_k \bar{\theta}_{k, a}) > 0 \text{ then } \\
\mu_{\alpha k}^{(h)}(n) = \min \{ \mu_{\text{max}}, \mu_{\alpha k}^{(h)}(n)(1 + \sum_{k \in X_{\text{brd}}} (\phi_k(n) - T_{\text{max}}(k) \Phi_{\text{brd}} \Delta t x_k \bar{\theta}_{k, a})) \} \\
\text{If } \sum_{k \in X_{\text{brd}}} (\phi_k(n) - T_{\text{max}}(k) \Phi_{\text{brd}} \Delta t x_k \bar{\theta}_{k, a}) \leq 0 \text{ then } \mu_{\alpha k}^{(h)}(n) = \mu_{\alpha k}^{(h-1)}(n)
\]
Figure 5: The general solution process
Plan of Presentation

- Introduction
- System Analysis
- Dynamic Network Modelling
- Performance indexes
- Global Optimization Problem
- Solution Approach
- Application
- Conclusions
<table>
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Exemple de programmation de vols
Organisation des flux dans le terminal
Flux incident et résultant au poste P₁

flux incident à P₁

flux résultant de P₁

$pax$
Arrivée des flux de passagers aux postes de check-in de XY
Evolution du niveau de service en XY
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CONCLUSIONS

This communication has introduced a general mesoscopic modelling approach for passengers flows in an airport terminal which appears to be compatible with the formulation of a relevant short term optimization problem for the allocation of available airlines and security staff.

The adopted network approach displays the dynamic interdependencies between the different flows and queuing systems while the degree of detail adopted allows the definition and quantification of detailed performance indexes.
An overall optimization problem has been formulated to meet security and comfort levels.

The excessive complexity of this optimization problem has led to propose its decomposition in two optimization subproblems:

- one devoted to the optimal assignment of airlines resources to check-in and boarding;

- one dedicated to the optimal assignment of passengers control resources.

A coordination process for the solutions of the two problems has been proposed. This process takes into account the predicted effects of decisions about the current sub-system over the others and by improving these predictions through repeated overall simulations. Also, an updating process is proposed to enforce the satisfaction of the overall service constraints.
Many questions related with the convergence of the overall process as well as with the quality of the obtained solution remain to be investigated. Different case studies should be considered to get more insight into these two questions.
Tank You for Your Attention

Questions?